International Journal of Occupational Safety and Ergonomics (*JOSE*) 2011, Vol. 17, No. 3, 271–276

Estimating Surface Acoustic Impedance With the Inverse Method

Janusz Piechowicz

Department of Mechanics and Vibroacoustics, AGH University of Science and Technology, Kraków, Poland

Sound field parameters are predicted with numerical methods in sound control systems, in acoustic designs of building and in sound field simulations. Those methods define the acoustic properties of surfaces, such as sound absorption coefficients or acoustic impedance, to determine boundary conditions. Several in situ measurement techniques were developed; one of them uses 2 microphones to measure direct and reflected sound over a planar test surface. Another approach is used in the inverse boundary elements method, in which estimating acoustic impedance of a surface is expressed as an inverse boundary problem. The boundary values can be found from multipoint sound pressure measurements in the interior of a room. This method can be applied to arbitrarily-shaped surfaces. This investigation is part of a research programme on using inverse methods in industrial room acoustics.

room acoustics   sound absorption coefficient   acoustic impedance

1. INTRODUCTION

Predicting sound field distribution or sound control tasks are typical problems of interior acoustics, in which numerical analysis methods, such as boundary elements or finite elements methods, are used. A description of boundary conditions in the form of the acoustic impedance (or admittance) of the surface is essential in the context of acoustic modelling. In 1919, Webster introduced the concept of acoustic impedance [1], an important quantity in studying acoustic elements and systems, and in particular when determining the acoustic properties of materials. Therefore, in addition to analytical methods of determining acoustic impedance, many measurement methods were developed, too. All measurement methods can be divided into three groups:

- surface methods;
- transmission methods;
  - moving microphone,
  - moving piston,
  - resonance,
- comparative methods.

Tabular values of impedance coefficients are not always sufficient for determining the distribution of sound field parameters consistently with values measured in an experiment. Discrepancies may result from the acoustic properties of wall coverings being different in actual conditions from values determined in the laboratory. In laboratory studies a normalized value of acoustic impedance is determined for material samples of sizes tailored to the dimensions of the impedance tube, by applying the standing wave coefficient method [2] or the transfer-function method [3].

Research and experiment studies were performed under the MNiSW N N 504 342536 research project at the AGH-UST Chair of Mechanics and Vibroacoustics.

Correspondence and requests for offprints should be sent to Janusz Piechowicz, Department of Mechanics and Vibroacoustics, AGH University of Science and Technology, Al. Mickiewicza 30, 30-059 Kraków, Poland. E-mail: <piechowi@agh.edu.pl>.

The differences between operating and laboratory conditions generate enhanced interest in methods that measure the absorption coefficient or acoustic impedance coefficients of the materials covering interior walls in operating (in situ) conditions.

2. DETERMINING ACOUSTIC IMPEDANCE IN IMPEDANCE TUBES

The values of surface impedance in laboratory conditions are determined for a material sample properly fastened at one end of a measuring impedance tube. The sound emitter (speaker) located at the other end of the tube emits a sinusoidal acoustic wave p_i which hits the sample. The superposition of plane waves, the incident p_i and reflected from the examined sample p_r, generates a standing wave in the tube, for which $p = p_i + p_r$. The measurement of amplitudes of acoustic pressure values is carried out in the pressure minima $|p(x_{min})|$ or maxima $|p(x_{max})|$ locations (one or more). These data are sufficient to determine the sound absorption coefficient α. To calculate the reflection coefficient R and the acoustic impedance Z it is necessary to determine the distance x_{min1} of the first acoustic pressure minimum position from the reference surface located at $x = 0$ (sample surface) and the wavelength value λ_o (Figure 1). The basic assumption of the method is that only plane waves propagate back and forth in the tube.

A plane, the harmonic acoustic wave p_i of the frequency f, propagates without damping and hits the sample surface plane [2]:

$$p_i(x) = p_0 e^{jk_0 x}$$
$$k_0 = \frac{2\pi f}{c_0}, \tag{1}$$

where p_0—arbitrary amplitude; c_0—sound velocity in air.

Equation 2 describes the wave reflected from the sample surface of the reflection coefficient R:

$$p_r(x) = Rp_0 e^{jk_0 x}. \tag{2}$$

The velocities of acoustic wave particles propagating in the positive and negative directions are, respectively,

$$v_i(x) = \frac{p_i(x)}{Z_0},$$
$$v_r(x) = -\frac{p_r(x)}{Z_0}, \tag{3}$$

where Z_0—characteristic acoustic impedance.

Equation 4 explains the acoustic impedance in the sound field of a standing wave:

$$Z(x) = \frac{p_i(x) + p_r(x)}{v_i(x) + v_r(x)} = Z_0 \frac{p_i(x) + p_r(x)}{p_i(x) - p_r(x)}. \tag{4}$$

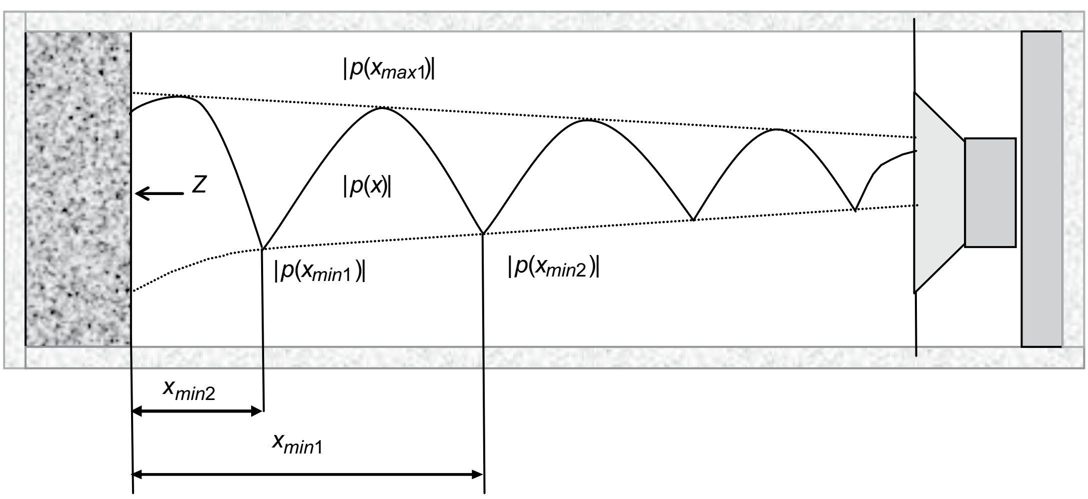

Figure 1. Standing acoustic wave in an impedance tube [2]. *Notes.* $|p_{xmax1}|$—maximum of sound pressure level for a given frequency; $|p_{min1}||p_{min2}|$—first and second minimum of sound pressure level for a given frequency; $|p(x)|$—sound pressure at x position for a given frequency; x_{min1}, x_{min2}—distance from the sample to first/second minimum of sound pressure level; Z—acoustic impedance of the sample.

Standard No. EN ISO 10534-2:2001 discusses determining acoustic impedance with the complex acoustic signal transfer function [3]. In that case, two measurement techniques are used: the dual-microphone method (two microphones in fixed positions) and the single-microphone method (one microphone placed consecutively in two positions). The complex acoustic transfer function determined in the impedance tube is

$$\boldsymbol{H}_{12} = \frac{S_{12}}{S_{11}} = |\boldsymbol{H}_{12}| e^{j\varphi} = \boldsymbol{H}_r + j\boldsymbol{H}_i, \qquad (5)$$

where S_{11}—autospectral density, S_{12}—cross spectral density; $\boldsymbol{H}_r$—real part of $\boldsymbol{H}_{12}$, $\boldsymbol{H}_i$—imaginary part of $\boldsymbol{H}_{12}$.

The complex reflection coefficient R for normal incidence of the acoustic wave is determined with Equation 6:

$$R = |R| e^{j\varphi} = \frac{\boldsymbol{H}_{12} - \boldsymbol{H}_l}{\boldsymbol{H}_R - \boldsymbol{H}_{12}} e^{2jkx}, \qquad (6)$$

where x—sample distance from the further microphone position, φ—phase angle of the reflection coefficient for normal incidence of the acoustic wave.

Equation 7 shows the actual value of the acoustic impedance Z calculated from the reflection coefficient R:

$$Z = Z_0 \frac{1+R}{1-R}. \qquad (7)$$

3. DETERMINING ACOUSTIC IMPEDANCE IN THE FREE FIELD

The dual-microphone method is a frequently used measurement method [4]. It implements measurements of direct sound from a speaker (one microphone) and sound reflected from a test surface (another microphone). The absorption coefficient of the examined material is calculated from the transfer function determined for each microphone location taking into account the location of the speaker. This method is recommended for large interiors, in which sound reflections from surrounding objects are negligible (free-field conditions). However, it unsuitable in anechoic chambers, in open-space conditions or in large-capacity halls, where sound reflections from neighboring surfaces can be eliminated from the pulse response function by selecting a proper time window.

Errors can in the dual-microphone method be expected when unwanted reflections cannot be eliminated from the registered signals or when the reflected wave is not a plane wave because the tested sample is small. The results can be also affected by the complexity of the shape of the examined surface.

In another method, the subtract technique, the microphone is located in the near vicinity of the examined surface. Pulsed responses of the incident and reflected waves are separated by a proper selection of the time window. The reference pulsed response signal is measured in free-field conditions. It is subtracted from the test measurement pulsed response signal [5]. The measurements are made for a frequency range from 250 Hz to 8 kHz, for the component normal to the plane surface of an area greater than 4 m^2. The results are usually inaccurate in the low-frequency range because of oblique reflections.

4. DETERMINING ACOUSTIC IMPEDANCE WITH THE INVERSE METHOD IN SITU

Inverse methods in vibroacoustics are practical in identifying sources of vibroacoustic energy, inverse problems of sound radiation by vibrating surfaces, and in evaluating acoustically working machinery by analysing sound field parameters. Inverse methods usually consist in creating an acoustic model and replacing the actual sound emitting source with a set of elementary sound sources. Then the acoustic parameters of these sources are simulated to obtain a sound field distribution produced by this system of substitute sources, which is as close as possible to the sound field distribution around the actual sound source [6, 7, 8].

There is some procedural analogy to determining acoustic impedance of the interior walls [9, 10]. For a sound field generated by a single acoustic source, the boundary element method provides approximate values of acoustic pressure that can be expected in the experimental study. The correctness of the respective numerical model for the Elmer[1] version 6.0 software (Finland) form the actual basis for formulating the inverse problem, determining acoustic impedance Z_S of an interior wall, with a given distribution of sound pressure values inside the examined space. The acoustic impedance Z_{Sx} at a given point x on the surface of the wall S is defined as

$$Z_{Sx} = \frac{p_{Sx}}{v_{Sx}}, \qquad (8)$$

where p_{Sx}—sound pressure at point x, v_{Sx}—acoustic particle velocity at point x.

Figure 2 presents the inverse problem of determining acoustic impedance for interior walls, with the sound source located on a vibrating part of the enclosed surface and the values of sound pressure known inside the enclosed space.

The dependence between the sound pressure p_m measured at an arbitrary point inside the examined space and the sound pressure p_S and acoustic particle velocity v_S at the boundary of this space in a steady-state sound field is given by the Helmholtz-Kirchhoff equation

$$\oint_S \left(p_s \frac{\partial G(r)}{\partial n} + j\omega\rho G(r) v_s \right) dS + p_m = 0, \qquad (9)$$

where $G(r)$—the Green function $G = \frac{e^{-jkr}}{4\pi r}$, $r = |r_m - r_s|$ is the distance between the point at the surface of the interior wall and the measurement point within the examined space, k—wave number, r—medium density.

The boundary surface area in the interior space model is divided into N equivalent elements S_i. Then sound pressure is measured in M measurement points inside the room. The model discretization by the boundary elements methods leads to a matrix equation:

$$\sum_{k=1}^{N} p_{sk} w_{ik} - \sum_{k=1}^{N} v_{sk} b_{ik} = -p_m, \qquad (10)$$

where $i = 1, 2, 3, \dots, M$.

Coefficients w_{ik} and b_{ik} are defined as

$$w_{i,l} = \int_{s_l} \frac{G(r)}{\partial n} ds,$$

$$b_{i,l} = -j\rho\omega \int_{s_l} G(r) ds,$$

where $i = 1, 2, \dots, M$; $l = 1, 2, \dots, N$.

Therefore, in the matrix form Equation 7 can be written as

$$W_m p_s - B_m v_s = -p_m, \qquad (11)$$

where p_s—sound pressure on the surface of the wall (a parameter of the space boundary),

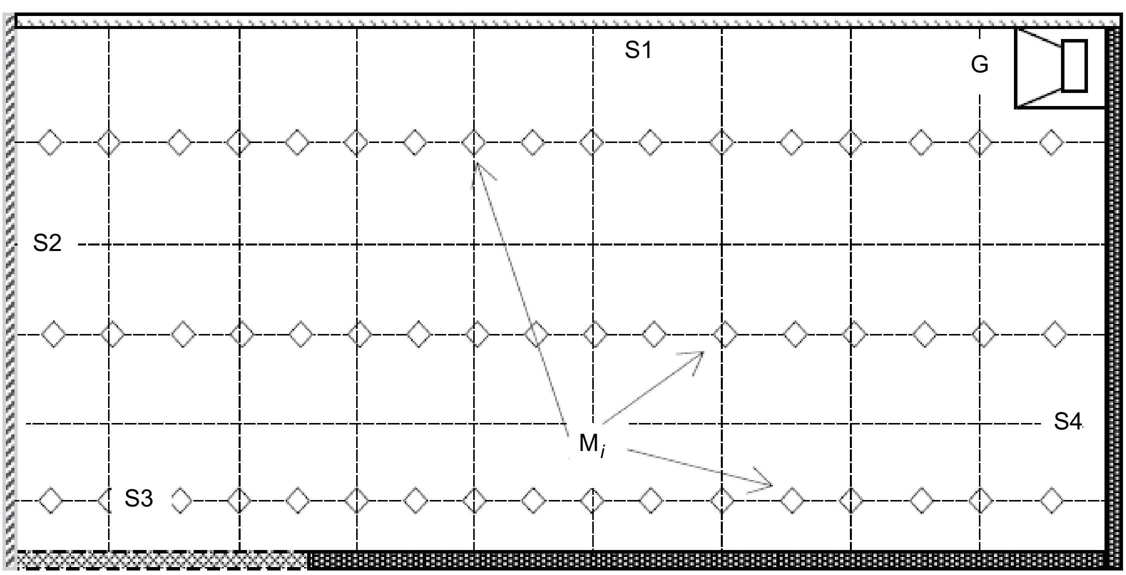

Figure 2. Application of the inverse boundary element method in interior acoustics. *Notes.* S1–S4—interior surfaces, M_i—measurement points (microphone locations), G—speaker.

[1] http://www.csc.fi/elmer

matrices W_m and B_m show the dependence between a point on the surface of the wall and a point for measuring sound pressure inside the examined space.

If the selected points for measuring sound pressure are located on the surface of the wall and if the node points are located on a smooth surface S, Equation 11 will take the form

$$W_s p_s - B_s v_s = 0, \quad (12)$$

where W_s, B_s—matrices of influence coefficients between two different points at the boundary surface *S*.

Therefore, for the whole system considered there is a set of equations [10] with unknown boundary parameters p_S and v_S:

$$W_s p_s - B_s v_s = 0, \; W_m p_s - B_m v_s = -p_m. \quad (13)$$

After substitutions,

$$W_m(W_s^{-1} B_s v_s) - B_m v_s = -p_m. \quad (14)$$

After transforming and substituting $C = (B_m - W_m W_s^{-1} B_s)$, the shortened form is

$$C v_S = p_m. \quad (15)$$

The *C* matrix coefficients depend only on the interior geometry and the frequency for which the analysis is carried out. According to Equation 15, after collecting a sufficient number of sound pressure measurements in points inside the room, it is possible to calculate the boundary parameter v_S and determine the acoustic impedance Z_S for the boundary surface enclosing the examined space. If it is impossible to determine the C^{-1} matrix, which is inverse to the *C* matrix, then the exact solution does not exist and the approximate values of v_S can be searched for by minimizing the mean-square error (the least squares problem) min $\|C v_S - p_m\|^2$.

Another solution of this problem can be sought by applying the singular value decomposition of the *C* matrix to $C = U \Sigma V^H$ [10]. After assuming that the *C* matrix has $m \times n$ dimensions, the *U* and *V* matrices are orthogonal and have dimensions $m \times m$ and $n \times n$, respectively, the dimensions of the Σ matrix are also $m \times n$. By using such an approximation of the *C* matrix, is possible to determine the inverse matrix by pseudoinversion of the *C* matrix, $C^+ = V \Sigma^{-1} U^H$. Therefore, Equation 12 can be presented as

$$v_S = C^+ p_m. \quad (16)$$

In real measurements, the measured value of sound pressure inside the room p_f is expected to contain, in addition to the sound field component from the sound source p_G, an additional noise component p_{ne}:

$$p_m = p_G + p_{ne}. \quad (17)$$

In some cases, the noise component may be even dominant. When using inverse methods to solve problems, it is necessary to use a solution regularization method.

5. CONCLUSION

Inverse methods can be effective in studying the acoustic parameters of working machinery in an industrial environment. They make it possible to evaluate acoustically machines on the basis of an analysis of the parameters of a sound field. The development of inverse methods in connection with other numerical methods, e.g., finite and boundary element methods or geometrical methods for determining acoustic signal propagation in interior spaces, makes locating sources for composite sound sources more efficient. Inverse methods are also better for determining acoustic parameters of an interior space. They can be used for practically all interior space geometries and do not require inconvenient and time-consuming numerical procedures. They are also suitable for reproducing vibration velocities of sound sources (radiation problems). The described method for determining acoustic impedance will be verified in the industrial environment. The study will be supported by measuring devices that allow multichannel (24 channels) and simultaneous measurements of sound pressure and phase shift of registered signals. Control over stimulation stability will make it possible to use the required measurement point density within the interior space.

REFERENCES

1. Webster AG. Acoustical impedance and theory of horns and of the phonograph. Proceedings of the National Academy of Sciences. 1919;5:275–82.
2. European Committee for Standardization (CEN). Acoustics—determination of sound absorption coefficient and impedance in impedances tubes—part 1: method using standing wave ratio (ISO 10534-1:1996) (Standard No. EN ISO 10534-1:2001). Brussels, Belgium: CEN; 2001.
3. European Committee for Standardization (CEN). Acoustics—determination of sound absorption coefficient and impedance in impedances tubes—part 2: transfer-function method (ISO 10534-2:1998) (Standard No. EN ISO 10534-2:2001). Brussels, Belgium: CEN; 2001.
4. Nocke C. In-situ acoustic impedance measurement using a free-field transfer function method. Appl Acoust. 2000; 59:253–64.
5. Mommertz E. Angle-dependent in-situ measurement of reflection coefficients using a subtraction technique. Appl Acoust. 1995;46(3):251–63.
6. Batko W, Dąbrowski Z, Engel Z, Kiciński J, Weyna S. Nowoczesne metody badania procesów wibroakustycznych cz. I [Modern methods of studying vibroacoustic processes part I]. Radom, Poland: ITE-PIB; 2005. In Polish.
7. Engel Z, Piechowicz J, Stryczniewicz L. Podstawy wibroakustyki przemysłowej [The principles of industrial vibro-acoustics]. Kraków, Poland: WIMiR AGH; 2003.
8. Piechowicz J. Determination of the sound power of a machine inside an industrial room by the inversion method. Arch Acoust. 2009;34(2):169–76.
9. Batko W, Dąbrowski Z, Engel Z, Kiciński J, Weyna S. Nowoczesne metody badania procesów wibroakustycznych cz. I [Modern methods of studying vibroacoustic processes part I]. Radom, Poland: ITE-PIB; 2005. In Polish.
10. Nava GP, Yasuda Y, Sakamoto S. On the situ estimation of surface acoustic impedance in interiors of arbitrary shape by acoustical inverse methods. Acoust Sci & Tech. 2009;30(2):100–9.