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"Slip and Fall" Theory—Extreme Order Statistics

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Classical "slip and fall" analysis was reformulated in this paper to account for the stochastic nature of friction. As it turned out, the new theory, arising from this analysis, was a precise statement of the distribution function for the smallest value among *n* independent observations. This made it possible to invoke an important result from the asymptotic theory of extreme order statistics that reduced the theory to a simple and elegant relationship among the probability of slipping, the critical friction criterion, the distance traveled by the walker, and the average, spread and asymmetry of the distribution of friction coefficients. The new theory reveals that short walks lead to fewer falls; low friction floors are sometimes better than high friction ones.

slip and fall   extreme value statistics   human locomotion   friction
slipperiness   Weibull   reliability

1. INTRODUCTION

The 1974 edition of *Accident Facts*, published by the National Safety Council, indicated that in the USA 8,000,000 falls occurred in the home resulting in 9,600 deaths and 1,600,000 disabling injuries[1]. In 1999, falling in the home once again killed 9,600; only motor vehicles caused more deaths than falling. The total number of deaths attributed to falls was 17,100 and the trend over the past 15 years is unfortunately increasing. Because of the seriousness of the "slip and fall" problem a great many technologists have focused on its elusive solution. Concentrating only on slips, there is

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[1] Death from accident is death that occurs within one year of the accident. Disabling injury is an injury causing permanent disability or any degree of temporary total disability.

general agreement that they may be abated by controlling the friction coefficients of floors.

Human locomotion involves acceleration during start-up, slowdown, steady movement, and maneuvers. These accelerations are associated with tangential forces transferred from a walker's footwear to the walking surface. To accomplish desired ambulation the tangential forces must be resisted by ground reaction forces. On uncontaminated dry floors, ground reaction forces are developed through friction.

In 1495 Leonardo da Vinci deduced the two basic laws of friction:

1. The friction force is dependent on the force pressing bodies together;
2. The friction force is independent of the apparent area of contact.

He found that the friction force was a fraction of the normal force, that is,

$$F = \mu N,$$

where F—friction force (tangential), μ—coefficient of friction (constant), N—normal component of the contact force between the contacting bodies.

Leonard Euler, in 1725, established that the coefficient of friction was different for static conditions, μ_s, and for dynamic or kinetic conditions, μ_k. He found that usually

$$\mu_s > \mu_k.$$

The static coefficient of friction is the ratio of horizontal force to normal force required to initiate sliding between two solid bodies. In 1875, Charles A. Coulomb discovered that kinetic friction, μ_k, is nearly independent of the sliding speed; this is often referred to as the third law of friction. These historical facts have been carefully chronicled by Duncan Dowson (1979) in his *History of Tribology*.

The required resistance for ambulation is measured with a force-plate. This is an instrumented walking surface that records the time history of contact forces impressed by walking candidates during various locomotion exercises, for example, straight walking or turning. Typical time histories are displayed in Figure 1, which has been generated from two sources by Grönqvist, Roine, Jarvinen, and Korhonen (1989). The top of the figure shows gait phases developed by Murray (1967) in normal level walking for one step with the right foot. The force-time diagrams in Figure 1 were obtained by Perkins (1978) using a three-axis force-plate manufactured by Kistler Instruments A.G. of Switzerland (Type 9261 A). Curves are shown for the horizontal force component H, the vertical force component V, and for their ratio H/V. The H/V ratio allows the limit of safety to be determined. The horizontal component of force applied by the foot to the floor is opposed by the friction between the two. At the point of slipping $H = \mu V$. Thus, if the ratio H/V is not as great as μ, slipping will not occur.

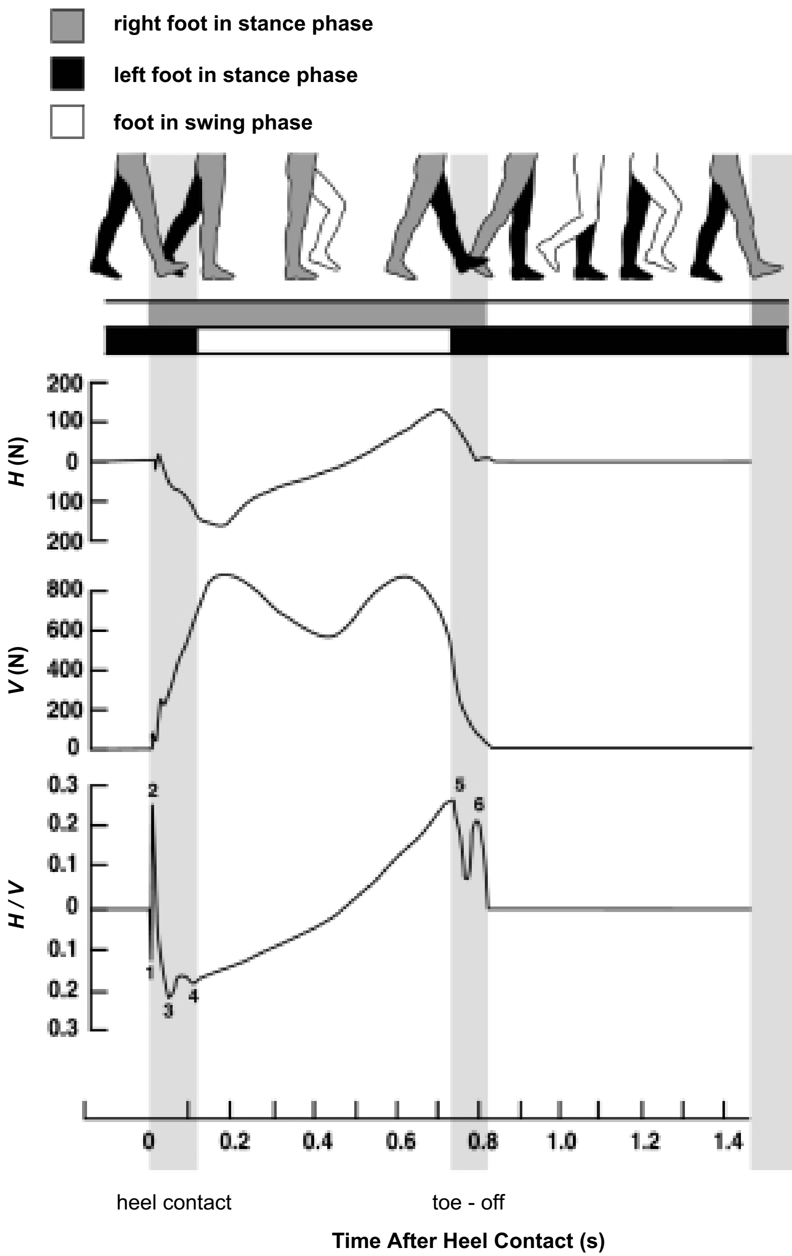

Figure 1. Contact forces during straight walking (after Grönqvist, Roine, Jarvinen, & Korhonen, 1989). *Notes.* Gait phases in normal level walking with typical horizontal force (H), vertical force (V), and their ratio (H/V) for one step (right foot). Critical from the viewpoint of slipping are the heel contact (especially peaks 3 and 4) and toe-off (peaks 5 and 6). N—Newtons.

The maximum value of *H/V* attained in a step will give the value below which μ, must not drop if the floor is to be safe.

In a comprehensive study by Harper, Warlow, and Clarke (1961), maximum values of *H/V* were determined on a level surface for men and women during straight walks and turns. Their force-plate measurements of *H/V*, which are summarized in Table 1, represent 87 sets of data for men and 37 sets for women. Using statistical inference, Harper et al. estimated the *H/V* value at the 0.9999 percentile level for straight walking, $H/V = 0.36$. This implies that only one in a million men will exceed this value. If there were such a thing as a uniform friction floor where the floor-footwear friction coefficient was everywhere constant, at say $\mu = 0.36$, then only one man in a million would slip on this surface. Unfortunately, the problem of slipping is more complicated because friction coefficients between material couples are stochastic. This paper deals with this fact.

TABLE 1. Maximum *H/V* (after Harper, Warlow, & Clarke, 1961)

			Turning			
	Straight Walking		Left Foot		Right Foot	
Statistical Properties	Men	Women	Men	Women	Men	Women
Mean (50 percentile)	0.17	0.16	0.19	0.17	0.22	0.19
Standard deviation	0.04	0.03	0.04	0.02	0.04	0.02
0.9999 percentile	0.36	—	0.40	—	0.36	—

2. CRITICAL FRICTION CRITERION

The classical formulation of the slip and fall problem may be stated as no slip will occur if the coefficient of friction μ between the walking surface and the footwear contact area exceeds some critical friction criterion, μ_c, that is,

$$\mu > \mu_c \text{ ... no slip.} \tag{1}$$

Equation 1 is deceptively simple. Almost every decision regarding the application of this inequality is surrounded by uncertainty; consider, in turn, the left and right sides:

Coefficient of Friction (Footwear/Walking Surface): μ

1. Should μ be taken as static or dynamic? For decades a transatlantic controversy has endured over this question; U.S. investigators have embraced the static coefficient of friction whereas the UK and European

experts have maintained that the kinetic friction coefficient is more significant.

2. Should a floor's slipperiness be judged under dry, wet, or oily conditions? To measure μ under wet or oily conditions a testing device should be selected that simultaneously applies the horizontal and vertical force components at the floor/footwear interface. This avoids erroneously high slip resistance values due to "sticktion;" this term was coined to express the adherence or build-up of suction at the test interface when the vertical force V is applied before H.
3. What footwear material should be used to test a floor; indeed, should an entire shoe be tested? Consider the following ASTM (American Society for Testing and Materials) standards:

   - ASTM C1028-89 (ASTM, 1989a) adopts Neolite,
   - ASTM D2047-88 (ASTM, 1988a) specifies leather conforming to Federal Specification KK-L-165C. Also a standard rubber may be used if it satisfies ASTM D1630 (ASTM, 1983a),
   - ASTM F609-89 (ASTM, 1989b) allows for actual footwear samples.

4. What slip-resistance measurement device should be used? There are as many as 40 kinds of tribometry appliances that have been developed in the past 70 years for laboratory and field applications under wet or dry conditions (Adler & Pierman, 1979; ASTM, 1983b, 1983c, 1983d, 1988b; Balance, Morgan, & Senior, 1985; Brungraber, 1976, 1977; Irvine, 1976; Jung & Schenk, 1990; Majcherczyk, 1977; Pfauth & Miller, 1976; Redfern & Bidanda, 1994; Reed & Mahon, 1977). The readings obtained among these devices are not consistent and they are not accurate when run head to head with force-plates. Most machines, on the other hand, show repeatability; both static and dynamic coefficients of friction lie within 10% of the mean (Andres & Chaffin, 1985).
5. How is the friction coefficient reported? When a particular test is executed, the result is a value of μ_s or μ_k. When the slipperiness of a floor is measured using a testing protocol, the average coefficient of friction is recorded, that is, $\bar{\mu}_s$ or $\bar{\mu}_k$. The following test protocols are examples:
   - ASTM C1028-89 (ASTM, 1989a), Horizontal Dynamometer Pull-Meter Method; 12 readings are averaged from four pulls perpendicular to the previous pull on each of three surface areas or three test specimens.
   - ASTM D2047-88 (ASTM, 1988a), James Machine; requires that the arithmetic average of 12 static coefficients of friction obtained from three panels be reported.

- ASTM Proposed Test method for "Dynamic Coefficient of Friction of Polish-Coated Floor Surfaces as Measured by the NBS-Sigler Pendulum Impact Tester (ASTM, 1983c); requires the arithmetic average of 24 readings of the dynamic coefficient of friction obtained on three specimens.
- ASTM Proposed Test Method for Static and Dynamic Coefficient of Friction of Polish-Coated Floor Surfaces as Measured by the Topaka Slip Tester (ASTM, 1983d); requires the arithmetic average of nine readings of static or dynamic friction coefficients taken from three specimens.

Classic floor slipperiness is always described as the average of multiple friction coefficient readings, that is, $\overline{\mu}_s$ or $\overline{\mu}_k$. For example, a high friction floor has a high average friction coefficient. In the present paper, friction coefficients are not only characterized by their average; but by their scatter and asymmetry as well.

6. Measuring friction:

- Every testing device has a protocol specified by the manufacturer or by associated standards such as those promulgated by ASTM or UL (Underwriters Laboratory).
- Seemingly minor things can have a significant effect on slipmeter readings; temperature, humidity, test foot material, test foot preparation, floor material, floor preparation, and the amount of time the two materials are in contact prior to attempting a test run. The temperature and humidity must be reported with the average friction coefficient for a number of different testing machines when ASTM procedures are adopted, for example, ASTM C1028-89 (ASTM, 1989a) and ASTM D2047-88 (ASTM, 1988a).

Critical Friction Coefficient: μ_c

1. The critical friction criterion μ_c is often established by legislative fiat. For example,
   - Australian-New Zealand Standard (Standards Australia and Standards New Zealand, 1993)
     Wet or Dry Horizontal Surfaces: When tested with the Pendulum Friction Tester (Wet) or the Floor Friction Tester (Dry) a pedestrian surface shall have a mean coefficient of friction of not less than 0.4 and no specimen in a sample (usually five specimens) shall be less than 0.35.

- Americans with Disabilities Act (ADA; Department of Justice, 2000)
  Accessible Routes: $\mu_c \geq \mu_s = 0.6$
  Ramps: $\mu_c \geq \mu_s = 0.8$
- Occupational Safety and Health Administration (OSHA, 2000):
  Ground and Floor Surfaces: $\mu_c \geq \mu_s = 0.5$.

It should be noted that no methods for determining the friction coefficients are specified by the ADA or OSHA.

2. For any particular tribometry device μ can be correlated with actual slipping experience and from these statistical data an acceptable μ_c may be chosen.
3. A minimum functional μ_c may be derived from locomotion analysis involving force-plates. A value judgement must be made regarding an acceptable number of slips; from this a certain percentile *H/V* may be selected as μ_c.
4. A safety factor may be applied to the average *H/V* obtained from force-plate studies to account for, among other things, variations in the coefficient of friction measurement. The resulting μ_c is sort of a hybrid of science driven by an experience factor.
5. The critical friction μ_c criterion may be arbitrarily taken as $\mu_c = \mu_s = 0.5$. This ubiquitous static friction coefficient enjoys a rich history dating back to 1945 and the James Machine of the Underwriters Laboratories ("Bucknell University F-13 workshop," 1992).

   Given these various methods of determining μ and establishing μ_c, one can hardly take the position that falling below μ_c necessarily leads to a slip or a slip and fall. Nevertheless, for shorthand purposes we shall define a slip as a violation of Equation 1.

3. REFORMULATION OF THE SLIP AND FALL PROBLEM

For a specific footwear material we may use one of the available tribometry machines to measure the coefficient of friction at various locations on a homogenous floor. The resulting set of data is called a statistical sample, which, in the usual way, may be presented as a histogram or as a cumulative distribution function such as that shown in Figure 2.

The term $F(\mu)$ is the probability that a random value of μ, M, is less than or equal to μ: $F(\mu) = P(M \leq \mu)$. Physically we know that μ cannot be less than zero; for a particular combination of materials it may never be less than μ_z, which we shall call the zero probability friction. The right hand side of the curve is shown to approach the value $F(\mu) = 1$ asymptotically. As a practical

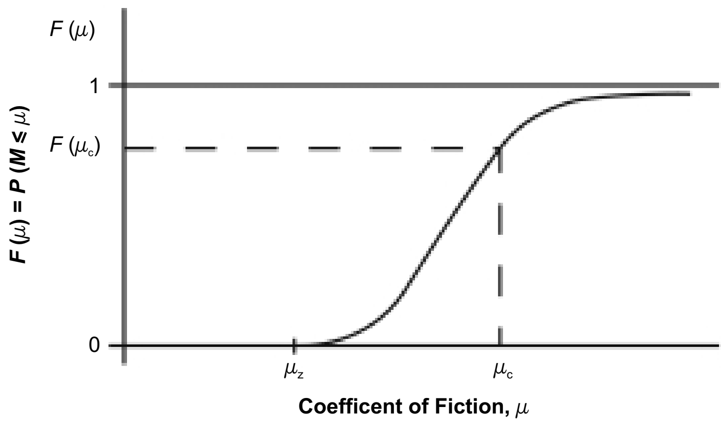

Figure 2. Cumulative distribution function: Coefficient of friction.

matter μ seldom exceeds unity; however, there is no theoretical reason precluding very large values. The development of friction resistance is related to the shape of the interface asperities. One can visualize intermeshing rigid square-tooth gear racks that produce sliding resistance without the imposition of normal forces. Macroscopically, child resistant bottle caps often use such a design for the clockwise or tightening direction.

Based on Equation 1, slipping proceeds whenever $\mu \leq \mu_c$. Consider taking a walk of n steps on a surface whose friction is characterized by the distribution function $F(\mu)$. During the first step the probability of slipping is $F(\mu_c)$ as indicated in Figure 2. On the other hand, the probability of surviving or not slipping is $[1 - F(\mu_c)]$. The survival of the second step is completely independent of the first step and has the same survival probability, $[1 - F(\mu_c)]$. Consequently, the probability of simultaneous survival of the first and second steps is the product $[1 - F(\mu_c)]\,[1 - F(\mu_c)]$. If we designate $F_w(\mu_c)$ as the probability of slipping during a walk of n steps, $[1 - F_w(\mu_c)]$ is the probability of surviving the walk. This is equal to the probability of simultaneously surviving n steps, $[1 - F(\mu_c)]^n$. Thus,

$$1 - F_w(\mu_c) = [1 - F(\mu_c)]^n \tag{2}$$

or

$$F_w(\mu_c) = 1 - [1 - F(\mu_c)]^n. \tag{3}$$

With reference to the Statistics of Extremes (Gumbel, 1958), Equation 3 turns out to be the definition of the exact distribution of the smallest value among n independent observations. We can now take advantage of some 85 years of mathematical inquiry in the field of extreme value statistics. In particular, the precise form of $F(\mu_c)$ may be obtained from the asymptotic theory of extreme order statistics and the observations that the friction coefficients are independent and identically distributed; they are continuously distributed and achieve a zero probability at $\mu = 0$ or $\mu = \mu_z$, and a walk of n steps follows the "weakest-link principle" in the sense that its resistance to slip cannot exceed the lowest friction coefficient encountered. $F(\mu_c)$ is a Weibull Distribution; that is,

$$
\begin{aligned}
F(\mu_c) &= 1 - e^{-\left(\frac{\mu_c - \mu_z}{\mu_o}\right)^m} && \ldots \mu_c \geq \mu_z \\
&= 0 && \ldots \mu_c \leq \mu_z,
\end{aligned}
\tag{4}
$$

where μ_z, μ_o, and m are statistical parameters. This result was first established in 1928 by Fisher and Tippett (1928); it is extensively explored in a remarkable book by Galambos (1978).

Substitution of Equation 4 into Equation 3 yields the principal finding of this study,

$$
\begin{aligned}
F_w(\mu_c) &= 1 - e^{-n\left(\frac{\mu_c - \mu_z}{\mu_o}\right)^m} && \ldots \mu_c \geq \mu_z \\
&= 0 && \ldots \mu_c \leq \mu_z.
\end{aligned}
\tag{5}
$$

The Weibull form is recaptured. This simple, elegant formula provides a relationship among the probability of slipping (or falling below μ_c), the length of the walk (n steps), the critical friction criterion μ_c and three statistical parameters that characterize the floor/footwear set. The three Weibull constants describe the entire distribution of friction coefficients including their average, their spread, and their asymmetry.

4. CHARACTERISTICS OF THE REFORMULATION

Because the implications of Equation 5 are so far-reaching and differ so radically from the classic slip and fall formulation, it seems appropriate to explore some of the important characteristics of this new theory.

4.1. Characterization—Floor/Footwear Set

In the classic slip and fall formulation, a floor/footwear set is characterized by establishing its average friction coefficient $\bar{\mu}$ through testing. In the new formulation, the distribution of friction coefficients is described by the Weibull distribution, or equivalently, by its three parameters μ_z, μ_o, and m. These parameters may be found by equating the first three moments of the Weibull distribution to the associated moments of the sample data obtained by testing.

The first moment of the Weibull distribution about the origin takes the form

$$\bar{\mu} = \mu_z + \mu_o \Gamma\left(1 + \frac{1}{m}\right), \tag{6}$$

where Γ is the gamma function. This represents a central measure of the distribution and displays the relationship among the sample mean $\bar{\mu}$ and the three Weibull parameters. This provides only one equation for three unknowns. The other two parameters may be obtained by computing the sample variance and skewness and relating them to their corresponding expressions using the central distribution moments, (about $\bar{\mu}$); thus,

$$s^2 = \mu_o^2\left[\Gamma\left(1 + \frac{2}{m}\right) - \Gamma^2\left(1 + \frac{1}{m}\right)\right], \tag{7}$$

where s^2 is the variance and s is the standard deviation, and

$$m_3/s^3 = \frac{\Gamma\left(1 + \frac{3}{m}\right) - 3\Gamma\left(1 + \frac{1}{m}\right)\Gamma\left(1 + \frac{2}{m}\right) - 2\Gamma^3\left(1 + \frac{1}{m}\right)}{\left[\Gamma\left(1 + \frac{2}{m}\right) - \Gamma^2\left(1 + \frac{1}{m}\right)\right]^{3/2}}, \tag{8}$$

where m_3 is the third central moment and m_3/s^3 is the skewness. Using Equations 6, 7, and 8 to solve for the Weibull parameters is a technique known as the method of moments; its usefulness is facilitated by a set of curves described by Gregory and Spruill (1962).

The reader should note that scatter is measured by the related concepts of variance, standard deviation, or coefficient of variation $s/\bar{\mu}$. The skewness is used as a measure of asymmetry. Frequency distributions such as $f(\mu)$ that show tails biased to the left, have negative skewness.

To illustrate the reformulation theory, a set of static friction coefficients were measured under laboratory conditions using a Horizontal Pull Slipmeter. Following the test protocol specified by ASTM F609-79 (ASTM, 1989b), 400 coefficients of friction were obtained between 100 new one-foot square asphalt tiles and three 0.5-inch (1.27-cm) diameter leather specimens under dry conditions. The sample data is presented as a histogram in Figure 3 and as a cumulative distribution function $F(\mu)$ in Figure 4. A continuous Weibull probability density curve $f(\mu)$ was fitted to the data in the histogram using

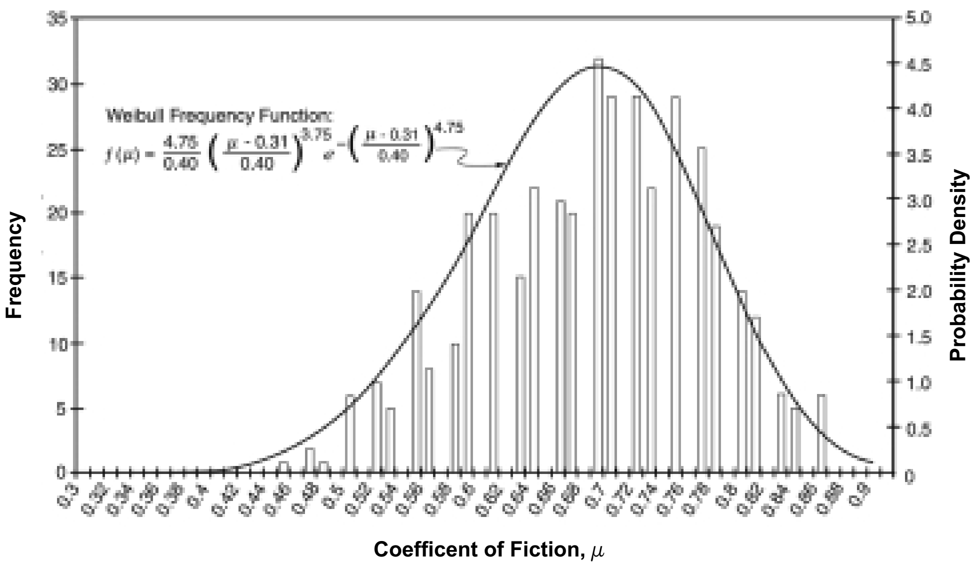

Figure 3. Histogram: Coefficient of friction.

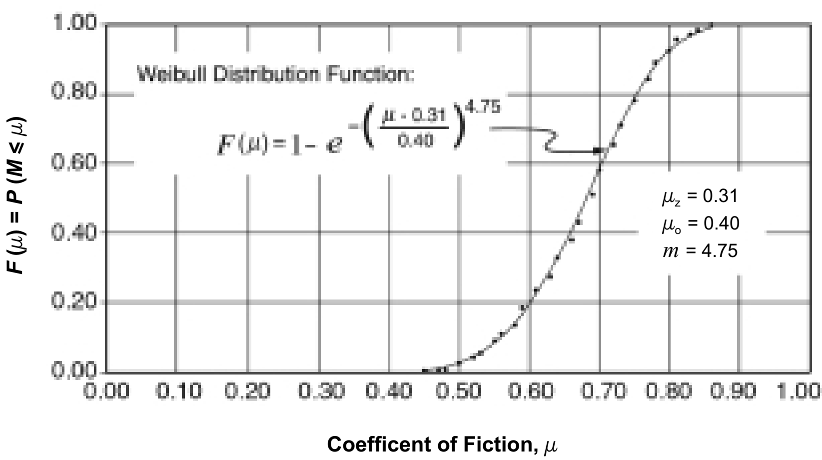

Figure 4. Cumulative distribution function: Coefficients of friction.

the parameters $\mu_z = 0.31$, $\mu_o = 0.40$, and $m = 4.75$. In Example 1, this data is used to illustrate the new slip theory in a thousand step walk of the type experienced in an airline terminal. For no particular reason the critical friction coefficient was taken as $\mu_c = 0.36$, which will preclude slipping for all but one man in a million.

Example 1

Weibull Parameters (from data):

$$\mu_z = 0.31$$
$$m = 4.75$$
$$\mu_o = 0.40$$

Length of walk:

$$n = 1000 \text{ ... } 1000 \text{ step walk}$$

Critical Friction Criterion:

$$\mu_c = 0.36$$

Mean (Average) Friction Coefficient ($\overline{\mu}$):

$$\begin{aligned}\overline{\mu} &= \mu_z + \mu_o \Gamma\left(1 + \frac{1}{m}\right) \\ &= 0.31 + (0.40)\,\Gamma\left(1 + \frac{1}{4.75}\right) \\ &= 0.676\end{aligned}$$

Variance (s^2):

$$\begin{aligned}s^2 &= \mu_o^2\left[\Gamma\left(1 + \frac{2}{m}\right) - \Gamma^2\left(1 + \frac{1}{m}\right)\right] \\ &= (0.4)^2\left[\Gamma\left(1 + \frac{2}{4.75}\right) - \Gamma^2\left(1 + \frac{1}{4.75}\right)\right] \\ &= 7.7232 \times 10^{-3}\end{aligned}$$

Standard Deviation (s):

$$s = 8.7882 \times 10^{-2}$$

Coefficient of Variation ($s/\overline{\mu}$):

$$\begin{aligned}s/\overline{\mu} &= 8.7882 \times 10^{-2} \div (0.676) \\ &= 13.00\%\end{aligned}$$

Skewness (m_3/s^3):

$$m_3/s^3 = \frac{\Gamma\left(1+\frac{3}{m}\right) - 3\Gamma\left(1+\frac{1}{m}\right)\Gamma\left(1+\frac{2}{m}\right) - 2\Gamma^3\left(1+\frac{1}{m}\right)}{\left[\Gamma\left(1+\frac{2}{m}\right) - \Gamma^3\left(1+\frac{1}{m}\right)\right]^{3/2}}$$

$$= \frac{\Gamma\left(1+\frac{3}{4.75}\right) - 3\Gamma\left(1+\frac{1}{4.75}\right)\Gamma\left(1+\frac{2}{4.75}\right) - 2\Gamma^3\left(1+\frac{1}{4.75}\right)}{\left[\Gamma\left(1+\frac{2}{4.75}\right) - \Gamma^3\left(1+\frac{1}{4.75}\right)\right]^{3/2}}$$

$$= -289.4768$$

Probability of slipping (or falling below μ_c), $F_w(\mu_c)$:

$$F_w(\mu_c) = 1 - e^{-n\left(\frac{\mu_c - \mu_z}{\mu_o}\right)^m}$$

$$F_w(0.36) = 1 - e^{-1000\left(\frac{0.36-0.31}{0.40}\right)^{4.75}}$$

$$= 5.0029\%$$

In conventional parlance, a tile floor of this composition will have a friction coefficient of 0.676 (actually, $\overline{\mu} = 0.676$), which is a high friction walking surface. Nevertheless, 5% of the walks will fail in the sense that a friction coefficient will be encountered that is less than 0.36.

4.2. Floor Slipperiness Ranking

The slipperiness of floors is conventionally ranked by their average coefficients of friction. This practice is rooted in the belief that “bigger is better,” that is, a higher friction floor leads to less slipping than a lower friction one. To demonstrate that this is not necessarily true, two additional examples of a thousand step walk are treated using the same critical friction criterion, $\mu_c = 0.36$, and the same zero probability friction, $\mu_z = 0.31$, adopted in Example 1.

Example 2 studies the properties of a floor/footwear couple with an average friction coefficient $\overline{\mu} = 0.5$, which is lower than the corresponding floor in Example 1 ($\overline{\mu} = 0.676$). Selecting an arbitrary value for the Weibull power parameter $m = 8$, the remaining parameter μ_o is calculated from Equation 6.

Example 2

Weibull Parameters:

$\mu_z = 0.31$ (determined from data)
$m = 8$ (assumed)
μ_o ... (to be calculated)

Length of Walk:

$n = 1000$ steps

Critical Friction Criterion:

$\mu_c = 0.36$

Mean Friction Coefficient:

$\overline{\mu} = 0.5$ (imposed)

Parameter, μ_o:

$$\mu_o = \frac{\overline{\mu} - \mu_z}{\Gamma\left(1 + \frac{1}{m}\right)} \quad \cdots \text{ from Equation 6}$$

$$= \frac{0.5 - 0.31}{\Gamma\left(1 + \frac{1}{8}\right)}$$

$$= 0.20175$$

Variance (s^2):

$s^2 = 7.9465 \times 10^{-4}$

Standard Deviation (s):

$s = 2.8190 \times 10^{-2}$

Coefficient of Variation ($s/\overline{\mu}$):

$s/\overline{\mu} = 5.64\%$

Skewness (m_3/s^3):

$m_3/s^3 = -1225.2386$

Probability of slipping (or <u>falling</u> below μ_c), $F_w(\mu_c)$:

$$F_w(0.36) = 1 - e^{-1000\left(\frac{0.36-0.31}{0.20175}\right)^{8}}$$

$$= 1.4131\%$$

We observe that the high friction floor/footwear set ($\bar{\mu} = 0.676$) presented in Example 1 gives rise to a slip probability $F_w(0.36) = 5\%$ whereas the lower friction floor/footwear set ($\bar{\mu} = 0.5$) provides a smaller slip probability $F_w(0.36) = 1.41$. It should be noted that the variance associated with the high friction set ($s^2 = 7.7232 \times 10^{-3}$) is an order of magnitude larger than the lower friction set ($s^2 = 7.9465 \times 10^{-4}$).

Comparing Examples 1 and 2 shows that here the scatter in the μ distribution is more important than the average $\bar{\mu}$. "Expressed in terms of floods, the statement is very simple: If a small stream has a larger dispersion of its discharges than a big river, it may cause larger floods than the big river" (Gumbel, 1954). These two examples not only force us to abandon our cherished classical notion that high friction is always better than low; but it teaches that floor slipperiness cannot be ranked according to average coefficients of friction.

Recall that the floor/footwear couple addressed in Example 1 used data obtained from clean dry asphalt tiles and leather specimens. Example 2 represents a study of an assumed floor/footwear couple with an average friction coefficient of $\bar{\mu} = 0.5$ and a coefficient of variation of 5.64%. Real world studies reported by Andres and Chaffin (1985) indicate that these friction characteristics are typical of those obtained between a painted cement floor in a large commercial laundry and rubber soled shoes and between a waxed cement floor in an automotive assembly plant and leather soled shoes.

In Examples 1 and 2, high average friction gave way to low scatter. This is not a general result as the following example will illustrate. The average friction in Example 3 is the same as that used in Example 2, $\bar{\mu} = 0.5$. The Weibull power parameter is taken as $m = 6$; Example 2 used $m = 8$. Once again, μ_o must be calculated using Equation 6.

Example 3

Weibull Parameters:

$\mu_z = 0.31$ (determined from data)
$m = 6$ (assumed)
μ_o ... (to be calculated)

Length of Walk:

$$n = 1000 \text{ steps}$$

Critical Friction Criterion:

$$\mu_c = 0.36$$

Mean Friction Coefficient:

$$\bar{\mu} = 0.5 \text{ (imposed)}$$

Parameter, μ_o:

$$\mu_o = \frac{\bar{\mu} - \mu_z}{\Gamma\left(1 + \frac{1}{m}\right)} \quad \cdots \text{ from Equation 6}$$

$$= \frac{0.5 - 0.31}{\Gamma\left(1 + \frac{1}{6}\right)}$$

$$= 0.2048 \qquad \cdots \text{ Weibull parameter}$$

Variance (s^2):

$$s^2 = 1.3652 \times 10^{-3}$$

Standard Deviation (s):

$$s = 3.6949 \times 10^{-2}$$

Coefficient of Variation ($s/\bar{\mu}$):

$$s/\bar{\mu} = 7.39\%$$

Skewness (m_3/s^3):

$$m_3/s^3 = -550.1373$$

Probability of slipping (or falling below μ_c), $F_w(\mu_c)$:

$$F_w(0.36) = 1 - e^{-1000\left(\frac{0.36 - 0.31}{0.20175}\right)^6}$$

$$= 19.0840\%$$

Example 3 shows a greater probability of slipping than Example 1 in spite of lower variability in the distribution of μ (coefficient of variation of 7.39% compared to 13.00%). Thus, a set with low average friction may exhibit a higher or lower probability of slipping compared to a set with higher average friction. The proper way to compare floor/footwear sets is to compare their $F_w(\mu_c)$'s using Equation 5.

Comparing Examples 2 and 3 whose floor/footwear sets have equal friction, $\overline{\mu} = 0.5$, we find that all other things being equal the lower scatter in Example 2 (coefficient of variation 5.64% compared to 7.39% in Example 3) leads to a smaller slipping probability. This result cannot be reached with conventional slip theory that would judge both floor/footwear couples to be equivalent. Once again, ranking by $\overline{\mu}$ is found to be impossible. Another provocative observation involves the application of floor treatments. A floor preparation that would not effect the average friction coefficient, may exert an inordinate influence on floor slipperiness by causing changes in the scatter and asymmetry of the friction coefficient distribution, $f(\mu)$. The effectiveness of floor treatments cannot be studied using conventional slip theory.

The reformulated theory of slip profoundly challenges existing tribometry appliances and the formulation of testing protocols. For example, the number of measurements required to estimate the mean friction coefficient of a floor/footwear couple is very small compared to the sample size required to reliably estimate the standard deviation and skewness. Given the poor track record of slipmeters for determining accurate estimates of the mean friction at high confidence levels, what chance do they have of reliably reflecting the variability and asymmetry of $f(\mu)$? The profound effect of these properties on slip probability is clearly established by the calculations associated with Examples 1, 2, and 3.

4.3. Length of Walk

The classic formulation of slip and fall is independent of the number of steps taken by the walker. In the new formulation, the probability of slipping may depend rather sensitively on the number of steps n. Consider the floor/footwear set described in Example 1. Taking values of μ_c as 0.33, 0.36, and 0.6, the probability of slipping is computed for various values of n as recorded in Table 2.

TABLE 2. Length of Walk (*n* Steps) Versus Probability of Slipping

n	F_w (0.33)	F_w (0.36)	F_w (0.6)
0	0	0	0
10	6.6085×10^{-6}	5.1311×10^{-4}	0.8859
100	6.6084×10^{-5}	5.1193×10^{-3}	1.0000
1000	6.6064×10^{-4}	5.0029×10^{-2}	1.0000
10,000	6.5868×10^{-3}	4.0145×10^{-1}	1.0000

Notes. Weibull parameters: $\mu_z = 0{,}31$, $m = 4.75$, $\mu_o = 0.4$

It is clear from this table that a floor cannot be characterized without including the number of anticipated steps n. Indeed, each time n increases by an order of magnitude, the corresponding slip probabilities $F_w(\mu_c = 0.33)$ and $F_w(\mu_c = 0.36)$ also increase by an order of magnitude. Furthermore, for long walks one finds that it is virtually certain that a friction coefficient will be encountered that is lower than $\mu_c = 0.6$. Keep in mind that this popular criterion, recommended by the Americans With Disabilities Act, already contains a safety factor. As a final observation, the slip probability is also very sensitive to changes in the critical friction criterion; increasing it by 10% from $\mu_c = 0.33$ to $\mu_c = 0.36$ increases the probability of slip by two orders of magnitude.

4.4. Infinite Walk

An examination of Equation 5 with a view toward taking the limit of F_w as n approaches infinity, indicates that whenever $\mu_c > \mu_z$ the quantity in parenthesis will be positive and the exponential term will always be driven to zero; thus, $F_w \to 1$ and slipping is certain to occur. To survive an infinite walk, μ_c must be less than or equal to μ_z which gives $F_w = 0$. It is unlikely that μ_c will ever equal μ_z given that μ_z is derived from friction data and μ_c has several definitions. Obviously, in an infinite walk one will encounter the lowest possible friction coefficient; if it is below the critical friction criterion slipping must occur by definition.

4.5. Selection of the Critical Friction Criterion

Rewriting Equation 5, an explicit expression for μ_c becomes,

$$\mu_c = \mu_z + \mu_o \left[\frac{-\ln(1 - F_w)}{n} \right]^{\frac{1}{m}} . \tag{9}$$

For a given length of walk and floor/footwear set everything in Equation 9 is known except the term $(1 - F_w)$, which is the subjective reliability that might be demanded by a value system.

4.6. Walking Profiles

Are several short walks more critical than a single equidistant long walk? Consider, for example, that k people take short walks of n steps each; the total number of steps is kn. The probability of slipping during a single short walk is given by Equation 5, that is, $1 - e^{-n\theta}$ where $\theta \equiv [(\mu_c - \mu_z)/\mu_o]^m$. As the short walks are all independent of each other, the total probability of slipping during the k walks is the sum of the k probabilities of slipping; $k(1 - e^{-n\theta})$. Now, consider a single equidistant walk of kn steps. The probability of slipping is $1 - e^{-kn\theta}$. The long walk may be treated as a series of walks; the associated slipping probabilities are not mutually exclusive as the first slip terminates the successful completion of the walk. Using the addition rule for arbitrary events the slipping probability for a $2n$ walk may be written for two n walks; thus,

$$(1 - e^{-2n\theta}) = (1 - e^{-n\theta}) + (1 - e^{-n\theta}) - (1 - e^{-n\theta})(1 - e^{-n\theta}).$$

Generalizing this result leads to the following identity:

$$(1 - e^{-kn\theta}) \equiv [1 - e^{-(k-1)n\theta}] + (1 - e^{-n\theta}) - (1 - e^{-n\theta})[1 - e^{-(k-1)n\theta}]. \qquad (10)$$

Evaluating and manipulating this equation for k's ranging from 2 to k leads to another identity; to wit,

$$(1 - e^{-kn\theta}) \equiv k(1 - e^{-n\theta}) - (1 - e^{-n\theta})\sum_{z=1}^{k-1}(1 - e^{-zn\theta}). \qquad (11)$$

Noting that all of the quantities in parentheses are non-negative probabilities, it is clear from Equation 11 that the slipping probability of a long walk, $1 - e^{-kn\theta}$, is always less than that of k short walks, $k(1 - e^{-n\theta})$. This implies that the number of steps n cannot be used to characterize a floor; the entire walking profile must be evaluated.

4.7. Central Tendency

Because the distribution of friction coefficients is skewed, the use of the mean to characterize a floor/footwear set has no physical significance. The mode, on the other hand, is the most likely value of μ that will be encountered. It may be expressed in terms of the Weibull parameters as

$$\mu_{\text{mode}} = \mu_{\text{z}} + \mu_{\text{o}} \left(\frac{m-1}{nm} \right)^{\frac{1}{m}} . \tag{12}$$

Another candidate for central tendency that retains its meaning for skewed distributions is the median: 50% of the friction coefficients are lower or higher than this value. Using Equation 9 with $F_{\text{w}} = 0.5$ gives

$$\mu_{\text{median}} = \mu_{\text{z}} + \mu_{\text{o}} \left(\frac{\ln 2}{n} \right)^{\frac{1}{m}} . \tag{13}$$

The expressions given in Equations 12 and 13 apply to the general Weibull form represented by Equation 5. When they are used to characterize the data used in Example 1, n is taken as unity; thus,

$$\mu_{\text{mean}} = 0.676,$$
$$\mu_{\text{mode}} = 0.691,$$
$$\mu_{\text{median}} = 0.680.$$

5. CONCLUSIONS

1. Conventional slip theory is formulated as a simple linear inequality relating the average friction coefficient and the critical friction criterion. It does not account for the stochastic nature of friction in either its characterization of the distribution of friction coefficients or in its applications in the area of human locomotion. By contrast, the new theory of slip and fall, Equation 5, provides a closed form and easily manipulated relationship among the probability of slipping, the critical friction criterion, the distance traveled by the walker, the average friction coefficient, and both the spread and asymmetry of the bell shaped friction distribution curve.
2. The average coefficient of friction for a floor/footwear set provides no information about the slipperiness of a floor. For the same average friction coefficient, the probability of slipping (or dropping below a given critical friction criterion) can vary widely as different values are attained for the standard deviation and skewness of $f(\mu)$ or for different walk lengths (n). In contrast with conventional slip theory, the new

formulation can be used to evaluate a floor, or rank various floor construction materials, or study the slipperiness of candidate floor preparations.

3. To evaluate the slip resistance [$F_w(\mu_c)$] of a floor system, the actual duty profiles must be established, for example, the number of walks of n_1 steps, n_2 steps, ... , n_i steps, that will be undertaken in a given time period.
4. To characterize floor slipperiness for a particular footwear material, a protocol must be available that can estimate the mean, standard deviation, and skewness of a friction coefficient sample with accuracy and confidence. This will require tribometers that can efficiently measure large numbers of friction coefficients. Furthermore, the machines must not effect a floor/footwear measurement by repeated trial reading at a single location. In addition, a slipmeter must suppress its own variability and that of its operator.
5. One of the most important findings that flows from the extreme value formulation of the slip problem is that friction data [$f(\mu)$] must follow the Weibull distribution.
6. The most important physical feature of the new formulation of the slip problem is the explicit recognition that slip depends on the smallest friction coefficient encountered during ambulation and not on the average friction $\overline{\mu}$ used in conventional theory.
7. The questions "how many walkers slip?" or "how many walkers fall?" are almost never the foci of an inquiry in the USA. Slip and fall analysis is almost entirely of the go/no go type, that is, the determination of compliance or non-compliance of the average friction coefficient with codes and standards arising from rule-making activities of consensus, statutory, or regulatory value systems. However these value systems arrive at a critical friction criterion μ_c, the new slip theory given by Equation 5 describes how often μ will drop below μ_c during a walking scenario. $F_w(\mu_c)$ never predicts the actual probability of slipping.

If μ_c is set equal to, say, the 50 percentile value of H/V for males during a straight walk, $\mu_c = 0.17$ (see Table 1), $F_w(0.17)$ predicts the probability that 50 percentile males will slip. This information is not enlightening.

If walkers do not slip, they will not fall. The converse is untrue; walkers that slip do not necessarily fall.

As it turns out, an appeal to reliability theory will enable us to calculate the probability of walkers actually slipping. The reliability R of a floor/footwear couple is the probability that its resistance μ_c is in excess of its loading (H/V) or that $\mu_c - (H/V) > 0$. In general, this reliability is given by Equation 14. This is the probability that walkers will not slip or fall, thus,

$$R = \int_{-\infty}^{\infty} \tilde{f}(H/V) \left[\int_{-\infty}^{H/V} f(\mu_c)\, d\mu_c \right] d(H/V) \,, \tag{14}$$

where $\tilde{f}(H/V)$ is a gaussian distribution expressed as

$$\tilde{f}(H/V) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left[\frac{H/V - \overline{H/V}}{\sigma}\right]^{½}} \,, \tag{15}$$

where $\overline{H/V}$ is the mean and σ is the standard deviation of the (H/V) distribution. In this representation, $-\infty \le H/V \le \infty$. Various sets of $(\overline{H/V}, \sigma)$ are given in Table 1. Also, the Weibull probability density function $f(\mu_c)$ may be found by differentiating Equation 5, that is,

$$f(\mu_c) = \frac{nm}{\mu_o}\left(\frac{\mu_c - \mu_z}{\mu_o}\right)^{m-1} e^{-n\left(\frac{\mu_c - \mu_z}{\mu_o}\right)^m} \quad \ldots\ \mu_c \ge \mu_z \ge 0 \tag{16}$$

$$= 0 \quad \cdots\ \mu_c \le \mu_z \ge 0 \,.$$

Equation 14 must be solved numerically. Kececioglu and Cormier (1964) give an excellent presentation on the development of R and they describe various methods for integrating it.

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